Cadastral Triangulation: A Block Adjustment Approach for Joining Numerous Cadastral Blocks

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INTRODUCTION

- Cadastre - method of land property registration and land parcels information
- Principles of Israeli cadastre (Torrens principles) - parcel boundaries are determined based on ground surveying
- Currently, the Israeli cadastre is based on block maps, mutation plans and parcel boundaries – marked on the ground and having legal validity

CADASTRAL CHARACTERISTICS

- Based on various geodetic control networks: in the past - Cassini-Soldner, Israeli TM, local systems; presently - IL2005 as a fully GPS based system
- Low accuracy of parcel points coordinates due to systematic errors of the geodetic control networks in the past
- Great difficulty in integrating adjoining blocks into a spatial cadastral continuity
- Not too many existing cadastral control points due to urban development activity and construction

POSSIBLE SOLUTION

- Transition from:
  the existing graphical based cadastre
- To
  a coordinate based cadastre
  optimal turning point positions
  improved accuracy

→→ Mathematical analytical/analogical adjustment procedures and processes

INTRODUCTION

A two stage solution:

- Developing an optimal method of original cadastral documents processing referring to separate projects – presented at FIGWW 2008

- Developing a model enabling joining the separate projects into a cadastral continuity of a rigid topological structure – current work

PROPOSED METHOD

Adopting principles of Aerial Triangulation:

- Separate aerial photographs are connected into blocks by common ***tie points*** in the overlapping regions of adjacent photographs
- Photogrammetric model is transformed from the model space to a ground coordinate system based on pre-defined ***control points***
- Transformation method - Block Adjustment by Independent Models

PROPOSED METHOD

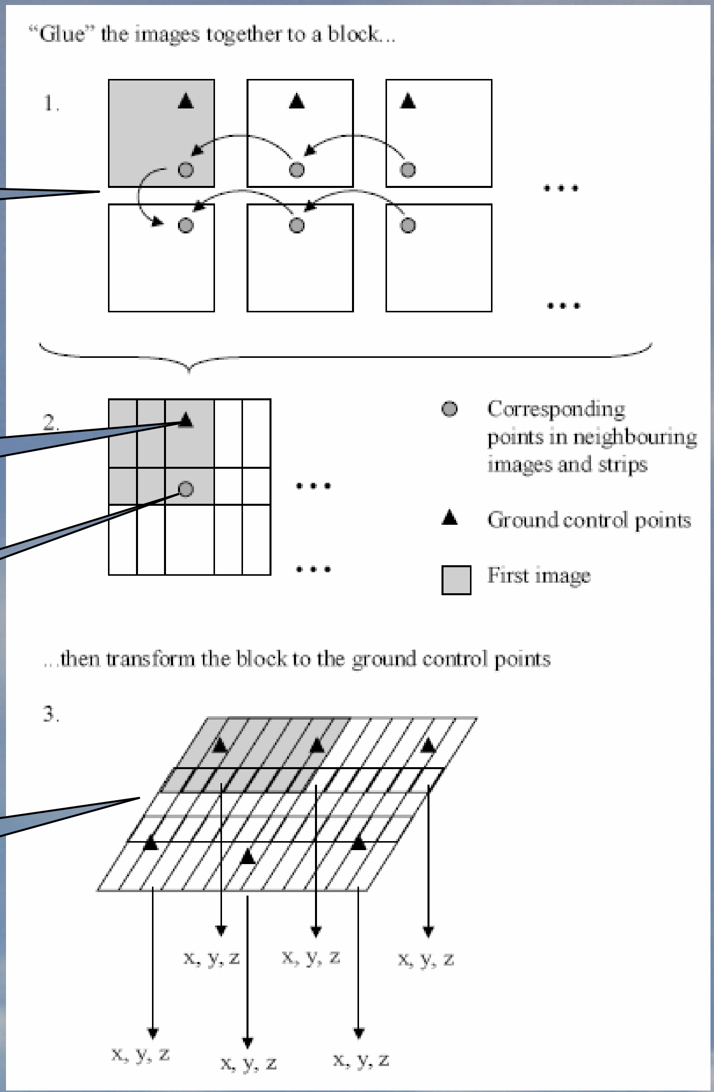
Aerial Triangulation →

Sequence of Aerial Photographs

Control Point

Tie Point

Block Adjustment and Transformation

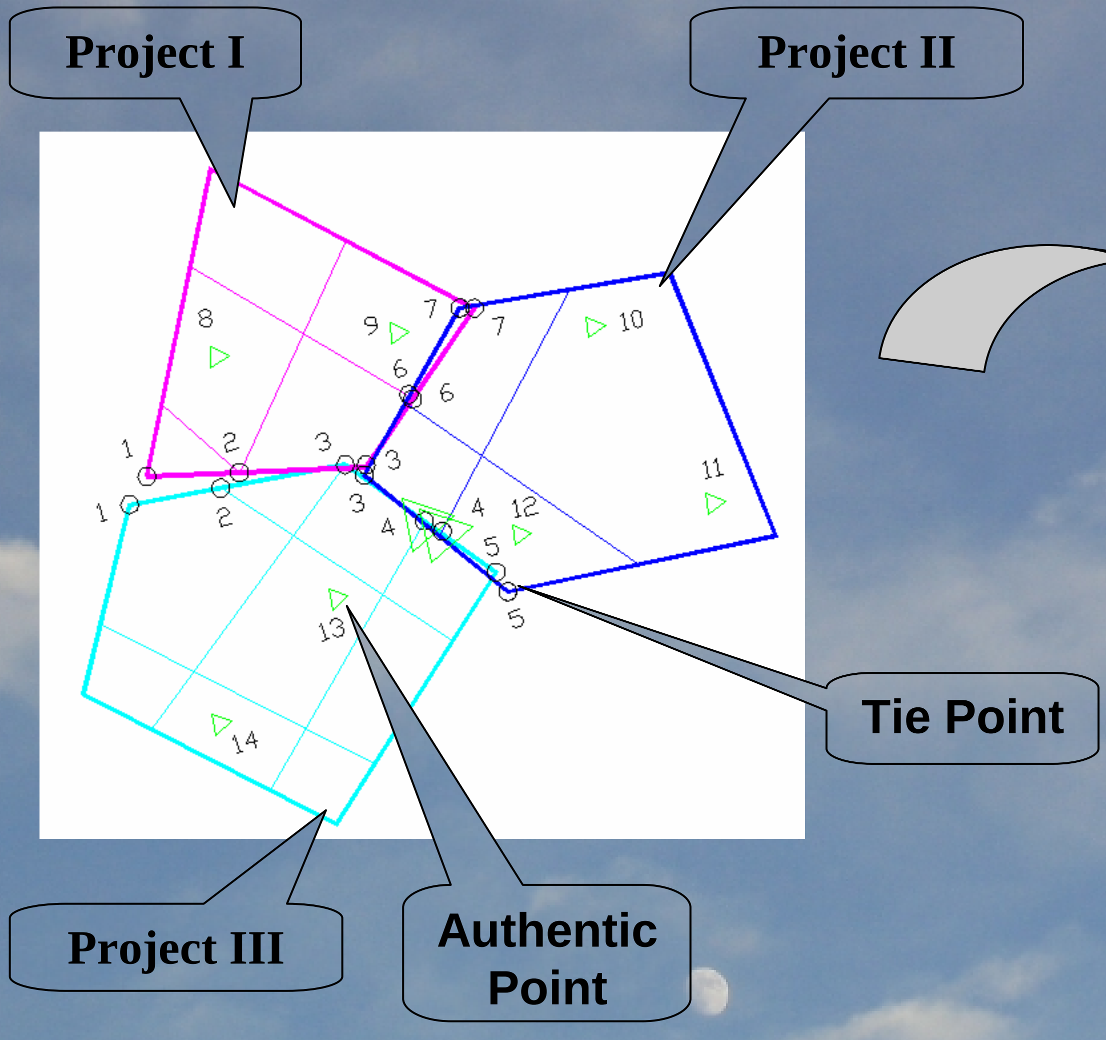


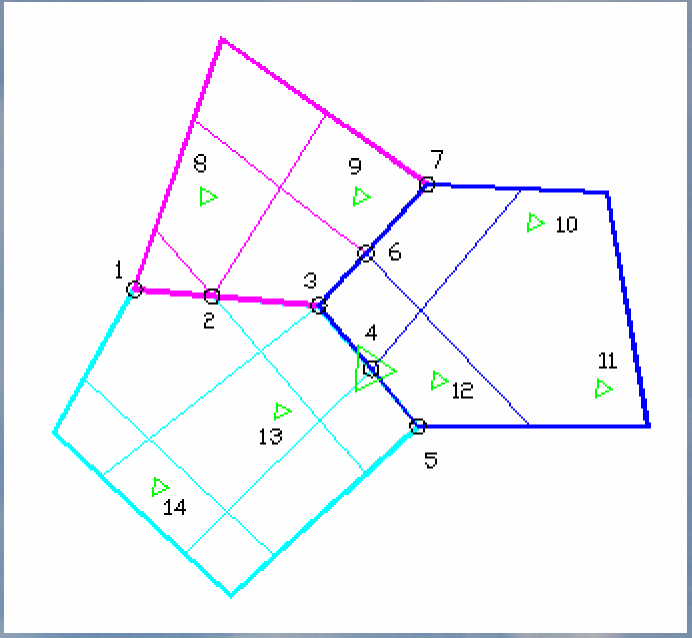
PROPOSED METHOD

Proposed Principles:

- Refers to separate cadastral projects (blocks and mutation plans) optimally pre-processed, determined in various coordinate systems (in their origin grids)
- ***Tie points*** - peripheral common turning points belonging to adjoining cadastral projects
- ***Control ("authentic“) points*** - cadastral project points that remained in the field and are re-measured (in the target grid)

PROPOSED METHOD





PROPOSED METHOD

Cadastral Triangulation (CT) Method:

- Global transformation of separate cadastral projects aiming to create a homogeneous seamless space by applying the Block Adjustment by Independent Models

- A Chained Similarity Transformation by applying the Least Squares Adjustment

MATHEMATICAL MODEL

Planar Similarity Transformation (linearized model)

$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} Y_o \\ X_o \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

where

- Y_t, X_t - point coordinates in the target grid
- Y_o, X_o - point coordinates in the original grid
- a, b - parameters of scale and rotation
- c, d - shift parameters

MATHEMATICAL MODEL

Chained Similarity Transformation

$$\left.\begin{aligned}\hat{\varepsilon}_{yi} &= \hat{a}^{j} * Y_{oi} - \hat{b}^{j} * X_{oi} + \hat{c}^{j} - \tilde{Y}_{ti} \\ \hat{\varepsilon}_{xi} &= \hat{a}^{j} * X_{oi} + \hat{b}^{j} * Y_{oi} + \hat{d}^{j} - \tilde{X}_{ti}\end{aligned}\right\} \Longrightarrow \left\{\begin{aligned}&\hat{\varepsilon} = X\hat{\beta} - y \\ &\sum p_i \hat{\varepsilon}_i^{\,2} = \min\end{aligned}\right.$$

where

- $\hat{\varepsilon}_i$ - residuals of point "i" computed from transformation parameters a,b,c,d of project "j"
- ~ - stands for coordinate estimates of tie points or known coordinates of authentic points in the target grid
- ^ - stands for Least Squares Estimator

MATHEMATICAL MODEL

Solution: LS Adjustment

$$\boxed{\hat{\beta} = (X^T P X)^{-1} X^T P y}$$

Fragments of Jacobian matrix "X", observations vector "y" and transposed vector of unknowns β^T for point "i" belonging to project "j":

$$(\beta^j)^T = [a^j \quad b^j \quad c^j \quad d^j]$$

for authentic point:

$$\left[X_i^j\right] = \begin{bmatrix} Y_{oi} & -X_{oi} & 1 & 0 \\ X_{oi} & Y_{oi} & 0 & 1 \end{bmatrix} \qquad [y_i] = \begin{bmatrix} Y_{ti} \\ X_{ti} \end{bmatrix}$$

$$(\beta^j)^T = [a^j \quad b^j \quad c^j \quad d^j \quad Y_{ti} \quad X_{ti}]$$

for tie point:

$$\left[X_i^j\right] = \begin{bmatrix} Y_{oi} & -X_{oi} & 1 & 0 & -1 & 0 \\ X_{oi} & Y_{oi} & 0 & 1 & 0 & -1 \end{bmatrix} \qquad [y_i] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

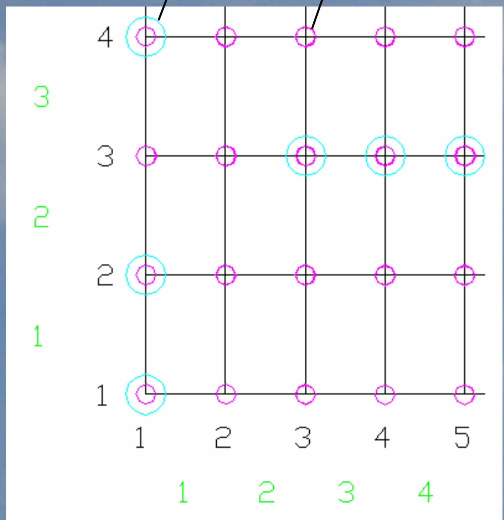
SIMULATION

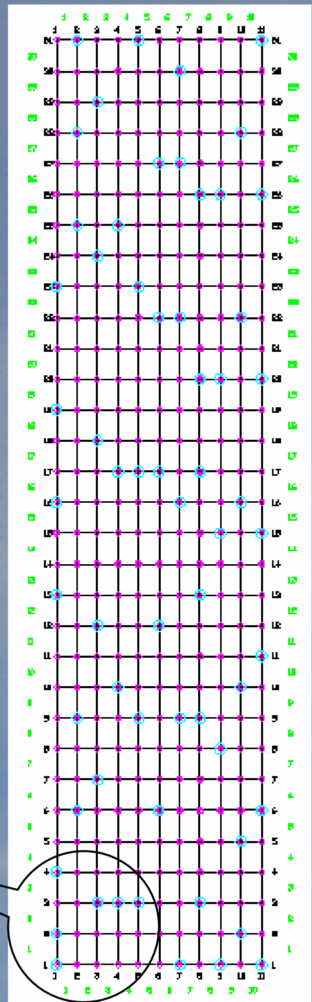
Ideal (original) situation

Array of 200x300 meter rectangles

control point

tie point





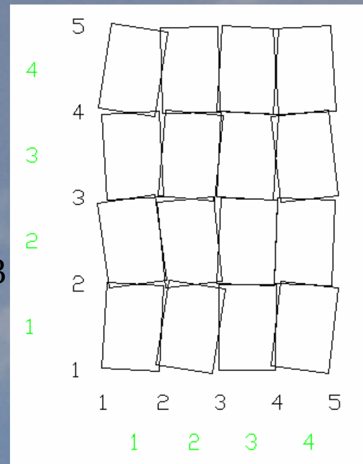
Distorted situation

Systematic errors of geodetic network

Scale: $\sigma_m = 0.0003$

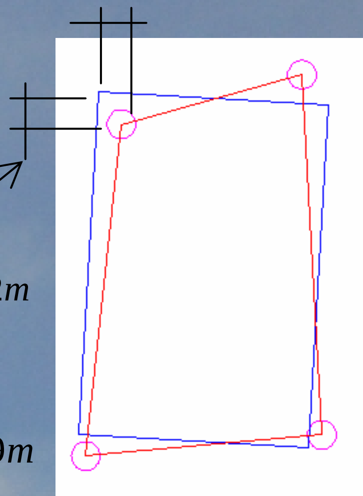
Rotation: $\sigma_K = 10^{\min}$

Shift: $\sigma_c = \sigma_d = 0.3m$



Random errors of ground surveying

Shift: $\sigma_{dY} = \sigma_{dX} = 0.12m$



Final StDev:

$\sigma_Y = \sigma_X = 0.49m$

SIMULATION RESULTS

Point position accuracy computed by the proposed method

Accuracy factor	Before adjustment (1)	After adjustment (2)	Improvement Ratio (1) vs. (2)
Sum of variances $\sum p_i \hat{\varepsilon}_i^{\,2}$ (sq. meters)	631.22	8.55	74
MSE $\sigma_0 = \sqrt{\frac{\sum p_i \hat{\varepsilon}_i^{\,2}}{r}}$ (meters)	0.51	0.08	6
Max residuals $\lvert \varepsilon_i \rvert$ (meters)	1.75	0.19	9

SIMULATION RESULTS

Differences between "true" coordinates and coordinates computed according to the proposed and currently used methods

Accuracy factor	"True" coordinates vs. proposed method coordinates (1)		"True" coordinates vs. current method coordinates (2)		Improvement Ratio (2) vs. (1)	
	Y	X	Y	X	Y	X
Average (meters)	0.00	0.00	-0.01	0.01	-	-
MSE (meters)	0.11	0.10	0.38	0.29	4	3
Max difference (meters)	0.36	0.33	1.03	1.02	3	3

SIMULATION RESULTS

Differences between "true" coordinates and coordinates computed according to proposed and currently used methods for reduced number of authentic points

Accuracy factor	"True" coordinates vs. **proposed method** coordinates (1)		"True" coordinates vs. **current method** coordinates (2)		Improvement Ratio (2) vs. (1)	
	Y	X	Y	X	Y	X
23 authentic points						
Average (meters)	-0.01	0.00	-0.04	0.03	-	-
MSE (meters)	0.14	0.14	0.44	0.34	3	2
12 authentic points						
Average (meters)	0.02	0.00	-0.01	0.01	-	-
MSE (meters)	0.17	0.20	0.43	0.34	3	2
6 authentic points						
Average (meters)	-0.04	-0.06	0.04	-0.02	-	-
MSE (meters)	0.32	0.32	0.42	0.33	1	1

CONCLUSION

Applying CT method enabled:

- Convert separate cadastral projects (blocks and mutation plans) prepared in various origin grids into cadastral continuity in a uniform geodetic target grid
- Reduce considerably position discrepancies between adjoining cadastral projects
- Increase position accuracy of parcel boundary turning points compared to the existing boundaries matching methods
- Obtain parcel turning points positions much nearer to their true positions compared to the existing boundaries matching methods

FUTURE WORK

An additional study should be made to analyze:

- Optimal number of transformation parameters referring to separate blocks during the global transformation and adjoining boundaries adjustment
- Optimal scattering of authentic points in the adjusted area; and the number authentic points
- Improvement of mathematical model to consider additional factors affecting final results
- Improvement of computational algorithm to deal with tens/hundreds of thousands of unknowns (when implementing the method on a nationwide scale)

Thank You

Questions ?