# APPLICATION OF A MULTI-PARAMETER TRANSFORMATION FOR DEFORMATION MONITORING OF A LARGE STRUCTURE 

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#### Abstract

A new methodology for deformation monitoring is applied to a large structure. The mathematical model for the new methodology utilizes a multi-parameter transformation relating original and repeated observations between an instrument station and any number of target points. The mathematical model is applied to original and repeated reflectorless total station observations made to target points in the roof of the Olympic Speedskating Oval in Calgary. (The Olympic Oval roof structure, with an unsupported roof span of approximately 80 m by 200 m , is one of the largest of its type in the world.) Results from this application indicate that the new methodology is very effective for deformation monitoring. Future work will include application of the new methodology to original and repeated three-dimensional laser scanner observations. The challenge with laser scanner observations (point clouds) is to match identical features in original and repeated point clouds. Recent research work in least squares orthogonal distance fitting of curves and surfaces in space may offer a solution to this problem.


## 1. Introduction

A new methodology for deformation monitoring is investigated by applying it to a large structure. The mathematical model for the new methodology is described in Section 2. Application of the new methodology, through an analysis of original and repeated reflectorless total station observations to target points on a large roof structure, is described in Section 3. A strategy for applying the new methodology to original and repeated threedimensional laser scanner observations, is outlined in Section 4.

## 2. Mathematical Model

The mathematical model for the new methodology utilizes an multi-parameter transformation relating original and repeated observations between an instrument station (e.g. total station or three-dimensional laser scanner) and any number of target points. The transformation consists of a 6-parameter similarity transformation at the instrument station (translations in the X-, Yand Z-directions at the instrument station, and rotations about the X-, Y- and Z-axes at the instrument station), plus a scale factor relating original and repeated instrument-target slope distance observations (or derived slope distance observations), plus a refraction correction between original and repeated zenith angle observations (or derived zenith angle observations).

This mathematical model can be expressed as follows:

$$
\begin{gather*}
\mathrm{X}_{\mathrm{O}}=\lambda\left(\mathrm{X}_{\mathrm{R}}+\kappa \mathrm{Y}_{\mathrm{R}}-\varphi \mathrm{Z}_{\mathrm{R}}\right)+\mathrm{T}_{\mathrm{x}}  \tag{1}\\
\mathrm{Y}_{\mathrm{O}}=\lambda\left(-\kappa \mathrm{X}_{\mathrm{R}}+\mathrm{Y}_{\mathrm{R}}+\omega \mathrm{Z}_{\mathrm{R}}\right)+\mathrm{T}_{\mathrm{y}}  \tag{2}\\
\mathrm{Z}_{\mathrm{O}}=\lambda\left(\varphi \mathrm{X}_{\mathrm{R}}-\omega \mathrm{Y}_{\mathrm{R}}+\mathrm{Z}_{\mathrm{R}}\right)+\mathrm{T}_{\mathrm{z}}  \tag{3}\\
\operatorname{with\mathrm {X}_{\mathrm {R}}}=\mathrm{S}_{\mathrm{R}} \sin H_{\mathrm{R}} \sin \left(\mathrm{~V}_{\mathrm{R}}+(\Delta \mathrm{R}) \mathrm{S}_{\mathrm{R}}\right)  \tag{4}\\
\mathrm{Y}_{\mathrm{R}}=\mathrm{S}_{\mathrm{R}} \cos H_{\mathrm{R}} \sin \left(\mathrm{~V}_{\mathrm{R}}+(\Delta \mathrm{R}) \mathrm{S}_{\mathrm{R}}\right)  \tag{5}\\
\mathrm{Z}_{\mathrm{R}}=\mathrm{S}_{\mathrm{R}} \cos \left(\mathrm{~V}_{\mathrm{R}}+(\Delta \mathrm{R}) \mathrm{S}_{\mathrm{R}}\right)  \tag{6}\\
\mathrm{X}_{\mathrm{O}}=\mathrm{S}_{\mathrm{O}} \sin \mathrm{H}_{\mathrm{O}} \sin \mathrm{~V}_{\mathrm{O}}  \tag{7}\\
\mathrm{Y}_{\mathrm{O}}=\mathrm{SO}_{\mathrm{O}} \cos H_{\mathrm{O}} \sin \mathrm{~V}_{\mathrm{O}}  \tag{8}\\
\mathrm{Z}_{\mathrm{O}}=\mathrm{S}_{\mathrm{O}} \cos \mathrm{~V}_{\mathrm{O}} \tag{9}
\end{gather*}
$$

in which $\mathrm{H}_{\mathrm{O}}, \mathrm{V}_{\mathrm{O}}$ and $\mathrm{S}_{\mathrm{O}}$ are original horizontal circle, vertical circle (zenith angle) and slope distance observations (or derived observations) respectively;
$H_{R}, V_{R}$ and $S_{R}$ are repeated horizontal circle, vertical circle (zenith angle) and slope distance observations (or derived observations) respectively;
$\mathrm{X}_{\mathrm{O}}, \mathrm{Y}_{\mathrm{O}}$ and $\mathrm{Z}_{\mathrm{O}}$ are X -, Y - and Z -coordinates computed from the original observations;
$X_{R}, Y_{R}$ and $Z_{R}$ are $X$-, $Y$ - and $Z$-coordinates computed from the repeated observations;
$\mathrm{T}_{\mathrm{x}}, \mathrm{T}_{\mathrm{y}}$ and $\mathrm{T}_{\mathrm{z}}$ are X -, Y - and Z -translations respectively at the instrument station; $\omega, \varphi$ and $\kappa$ are rotations about the $\mathrm{X}-, \mathrm{Y}$ - and Z -axes respectively at the instrument station;
$\lambda$ is the scale factor relating original and repeated slope distance observations; and ,
$\Delta \mathrm{R}$ is the refraction correction (in arc seconds per metre of slope distance; see reference [6]) relating original and repeated zenith angle observations (or derived observations).
The set of equations (1) through (9) inclusive can be solved as an implicit nonlinear least squares adjustment to obtain the transformation parameters $\omega, \varphi, \kappa, T_{x}, T_{y}, T_{z}, \lambda$ and $\Delta R$; corrected observations $\mathrm{H}_{\mathrm{O}}, \mathrm{V}_{\mathrm{O}}, \mathrm{S}_{\mathrm{O}}, \mathrm{H}_{\mathrm{R}}, \mathrm{V}_{\mathrm{R}}$ and $\mathrm{S}_{\mathrm{R}}$ to each target point; and movements ( $\mathrm{X}_{\mathrm{T}}$ -$\left.\mathrm{X}_{\mathrm{O}}\right),\left(\mathrm{Y}_{\mathrm{T}}-\mathrm{Y}_{\mathrm{O}}\right)$ and $\left(\mathrm{Z}_{\mathrm{T}}-\mathrm{Z}_{\mathrm{O}}\right)$ of each target point. $\left(\mathrm{X}_{\mathrm{T}}, \mathrm{Y}_{\mathrm{T}}\right.$ and $\mathrm{Z}_{\mathrm{T}}$ are transformed X -, Y - and Z-coordinates as given by the right-hand-sides of Equations (1), (2) and (3) respectively.)

## 3. Application of the Multi-Parameter Transformation: Olympic Oval Roof

### 3.1 Background

The Olympic Speedskating Oval in Calgary is a very large, uniquely designed structure. It was built for the 1988 Winter Olympics. The Olympic Oval roof structure, with an
unsupported span of approximately 80 m by 200 m , is one of the largest of its type in the world.

The roof structure of the Olympic Oval consists of 84 interconnected hollow-core beam columns. The external cross section of the beam columns is approximately 1 m wide by 2 m deep. The roof structure is hinged at the tops of buttressed columns, with both the columns and buttresses founded on concrete piles. The columns are approximately 1.5 m in diameter and the buttresses are approximately 1.5 m wide by 2 m deep. Fig. 1(a) shows a cross section through the Olympic Oval and Fig. 1(b) shows the west elevation.


Figure 1 - Olympic Oval, Calgary

The Olympic Oval has experienced both short-term and long-term deformations. The shortterm deformations (deformations occurring as soon as the load is applied) are due to:

1. Dead weight load of the structure itself.
2. Snow load on the structure.
3. Wind load on the structure.
4. Temperature changes in the structure.

The long-term deformations are due to:

1. Shrinkage of the concrete.
2. Creep of the concrete and soil (progressively smaller deformations occurring over a period of time under constant loading conditions).
3. Changes in soil stiffness because of variations in moisture content of the soil.

The first deformations of interest (those due to the dead weight load of the roof structure) occurred when the roof structure was lowered onto the buttressed column substructure in June 1986. An analysis of these deformations is summarized in [7]. Deformations of interest which occurred after the June 1986 dead weight load deformations were those due to creep and shrinkage of the concrete in the roof beam columns. An analysis of these deformations is also summarized in [7].

### 3.2 Deformation Monitoring

In recent years, the only significant deformations in the Olympic Oval are vertical movements of the roof structure caused by seasonal temperature variations. A detailed analysis of these deformations is given in [7].


Figure 2 - Plan View of Olympic Oval Roof

Based on the known seasonal movement of the Olympic Oval roof structure, it was planned to apply the multi-parameter transformation to an epoch of original observations made in July 2005 (outside temperature about +30 degrees Celcius) and an epoch of repeated observations made in January 2006 (outside temperature about -30 degrees Celcius). Unfortunately, January 2006 and the first two weeks of February 2006 were unseasonably warm. It was therefore decided to apply the multi-parameter transformation to two other epochs of observations (original and repeated), one made on July 4, 2005 and the other made on July 6, 2005.

A small subset of original and repeated reflectorless total station observations are shown in Table 1, with horizontal circle observations denoted as H, vertical circle observations denoted
as V , and slope distance observations denoted as S . These observations were made from the floor of the Olympic Oval at point 201 to roof points 5, 10, 15, 20, 25 and 30; see Figure 2. The measurements were made with a Leica TCR 803 reflectorless total station. Estimated standard deviations of the total station observations are $+/-2$ arc seconds for horizontal and vertical circle observations, and $+/-2 \mathrm{~mm}$ for slope distance observations.

| Roof Point | H(dms) | V(dms) | S(m) |
| :---: | :---: | :---: | :---: |
| 5/Original | 351-28-49 | 80-48-41 | 87.301 |
| 5/Repeated | 351-28-58 | 80-47-19 | 87.306 |
| 10/Original | 348-59-35 | 75-20-55 | 74.825 |
| 10/Repeated | 348-59-47 | 75-19-24 | 74.834 |
| 15/Original | 344-45-39 | 68-19-00 | 56.956 |
| 15/Repeated | 344-45-48 | 68-17-01 | 56.969 |
| 20/Original | 337-09-55 | 60-41-26 | 42.816 |
| 20/Repeated | 337-10-07 | 60-39-04 | 42.833 |
| 25/Original | 314-51-44 | 46-47-26 | 30.629 |
| 25/Repeated | 314-51-50 | 46-44-31 | 30.653 |
| 30/Original | 260-28-12 | 38-41-27 | 26.858 |
| 30/Repeated | 260-28-16 | 38-38-44 | 26.885 |

Table 1 - Reflectorless Total Station Observations to Olympic Oval Roof Points

### 3.3 Analysis and Results

The mathematical model described in Section 2 was used to recover the deformations. In this application, rotations $\omega$ and $\varphi$ were set to zero because the total station has dual axis compensation. Translations $\mathrm{T}_{\mathrm{x}}$ and $\mathrm{T}_{\mathrm{y}}$ were also set to zero because the total station was centered over the same point for original and repeated observations. Scale factor $\lambda$ and refraction correction $\Delta \mathrm{R}$ were dealt with as free parameters since one could reasonably expect different atmospheric conditions on July 4 and July 6 . Rotation $\kappa$ was dealt with as a free parameter to allow rotation in the horizontal plane.

The results from the application of the mathematical model to recover movements are shown in Tables $2 \mathrm{a}, 2 \mathrm{~b}$ and 2 c . These results show that the movement of the roof points (zero movement) was recovered by the mathematical model described in Section 2.

| Location | Movement(mm) | Stn Dev(mm) | Significant ? |
| :---: | :---: | :---: | :---: |
| Roof Point 5 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=+\mathbf{0 . 2}$ | $+/-0.3$ | No |
| Roof Point 10 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=-\mathbf{0 . 3}$ | $+/-0.3$ | No |
| Roof Point 15 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=+0.4$ | $+/-0.3$ | No |
| Roof Point 20 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=-0.3$ | $+/-0.2$ | No |
| Roof Point 25 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=+0.3$ | $+/-0.3$ | No |
| Roof Point 30 | $\left(\mathbf{Z}_{\mathbf{T}}-\mathbf{Z}_{\mathbf{O}}\right)=-0.3$ | $+/-0.3$ | No |

Table 2a - Recovered Z-Movements, Olympic Oval

| Location | Movement(mm) | Stn Dev(mm) | Significant ? |
| :---: | :---: | :---: | :---: |
| Roof Point 5 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=+\mathbf{0 . 1}$ | $+/-\mathbf{0 . 2}$ | No |
| Roof Point 10 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=+\mathbf{0 . 1}$ | $+/-0.2$ | No |
| Roof Point 15 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=-\mathbf{0 . 4}$ | $+/-0.2$ | No |
| Roof Point 20 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=+\mathbf{0 . 1}$ | $+/-0.2$ | No |
| Roof Point 25 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=+\mathbf{0 . 1}$ | $+/-0.2$ | No |
| Roof Point 30 | $\left(\mathbf{X}_{\mathbf{T}}-\mathbf{X}_{\mathbf{O}}\right)=+\mathbf{0 . 1}$ | $+/-0.2$ | No |

Table 2b - Recovered X-Movements, Olympic Oval

| Location | Movement(mm) | Stn Dev(mm) | Significant ? |
| :---: | :---: | :---: | :---: |
| Roof Point 5 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=+\mathbf{0 . 2}$ | $+/-\mathbf{0 . 4}$ | No |
| Roof Point 10 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=-\mathbf{0 . 3}$ | $+/-0.4$ | No |
| Roof Point 15 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=+\mathbf{0 . 4}$ | $+/-0.4$ | No |
| Roof Point 20 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=-0.3$ | $+/-0.3$ | No |
| Roof Point 25 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=+\mathbf{0 . 3}$ | $+/-0.3$ | No |
| Roof Point 30 | $\left(\mathbf{Y}_{\mathbf{T}}-\mathbf{Y}_{\mathbf{O}}\right)=-\mathbf{0 . 3}$ | $+/-0.3$ | No |

Table 2c - Recovered Y-Movements, Olympic Oval

## 4. Strategy for Applying Multi-Parameter Transformation to Laser Scanner <br> Observations

### 4.1 Work to Date

Results from the application of the multi-parameter transformation to planned threedimensional laser scanner observations in a typical industrial survey application (see Figure 3) are shown in Table 3 and reported in [8]. Estimated standard deviations of laser scanner observations ( $+/-5$ arc seconds and $+/-1 \mathrm{~mm}$ ) are given in [4] and [5].
The problem with this analysis is that points R1 to R18 inclusive in Figure 3 must be circular or spherical target points. In an actual application it would be far too time-consuming to set these points each time observations were made.


Figure 3 - Plan View of Location of Target Points at Indoor Industrial Site

| Location | Standard Deviation of Movement(mm or arcsecs) | Standard Deviation of Mean Position(mm) |
| :---: | :---: | :---: |
| Machine Point M1 | ( $\mathrm{X}_{\mathrm{T}}-\mathrm{X}_{\mathrm{O}}$ ): +/- $\mathbf{0 . 1 4}$ | $\left(\mathrm{X}_{\mathrm{T}}+\mathrm{X}_{\mathrm{O}}\right) / 2 \mathrm{~L}$ +/-0.07 |
| Machine Point M1 | ( $\mathrm{Y}_{\mathrm{T}}-\mathrm{Y}_{\mathrm{O}}$ ): $\mathrm{+} /-\mathrm{0.24}$ | $\left(\mathrm{Y}_{T}+\mathrm{Y}_{\mathbf{O}}\right) / \mathbf{2}:+/-\mathbf{0 . 1 2}$ |
| Machine Point M1 | $\left(\mathrm{Z}_{\mathrm{T}}-\mathrm{Z}_{\mathrm{O}}\right):+/-\mathbf{0 . 3 6}$ | $\left(\mathrm{Z}_{\mathrm{T}}+\mathrm{Z}_{\mathrm{O}}\right) / 2:+/-0.18$ |
| Machine Point M2 | ( $\mathrm{X}_{\mathrm{T}}-\mathrm{X}_{\mathrm{O}}$ ): $+/-0.14$ | $\left(\mathrm{X}_{\mathrm{T}}+\mathrm{X}_{\mathbf{O}}\right) / \mathbf{2}:+/-0.07$ |
| Machine Point M2 | ( $\mathrm{Y}_{\mathrm{T}}-\mathrm{Y}_{\mathrm{O}}$ ) : +/-0.24 | $\left(\mathrm{Y}_{\mathbf{T}}+\mathrm{Y}_{\mathbf{O}}\right) / \mathbf{2}:+/-\mathbf{0 . 1 2}$ |
| Machine Point M2 | $\left(\mathrm{Z}_{\mathrm{T}}-\mathrm{Z}_{\mathrm{O}}\right):+/-0.36$ | $\left(\mathrm{Z}_{\mathrm{T}}+\mathrm{Z}_{\mathrm{O}}\right) / 2:+/-0.18$ |
| Laser Scanner | $\omega:+/-2.6$ | --- |
| Laser Scanner | $\varphi:+/-5.3$ | --- |
| Laser Scanner | к: +/-2.7 | --- |
| Laser Scanner | $\mathrm{T}_{\mathrm{x}}$ : +/- 0.07 | --- |
| Laser Scanner | $\mathrm{T}_{\mathrm{y}} \mathrm{t}$ +/-0.12 | --- |
| Laser Scanner | $\mathrm{T}_{\mathrm{z}}$ : +/- 0.18 | --- |

Table 3 - Standard Deviations of Movements and Mean Positions,
Indoor Industrial Site

### 4.2 Strategy

The strategy to overcome the problem described in Section 4.1 is to match identical features in original and repeated laser scanner observations (point clouds). For man-made structures, the intersection of three or more angular surfaces might work well [3]. For natural structures, generation of virtual points of maximum curvature may be the only alternative. Recent research work in least squares orthogonal distance fitting of surfaces in space [1] [2] provides information on how virtual points of identical natural features might be generated.

## 5. Conclusion

The results indicate that the new methodology is very effective for deformation monitoring utilizing reflectorless total station observations. The new methodology for deformation monitoring should also be very effective utilizing three-dimensional laser scanner observations, once some interesting technical challenges are overcome.

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