

The Algorithm of Urban Estates Valuation

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Urban Real Estates

- Urban real estates definition
- Specific parameters
- The estates' prices registration in Poland

VALUATION MODEL PARAMETERS' ESTIMATION

$$C_{Tj} = S_1 \times c_1 + S_2 \times c_2 + \dots + S_i \times c_i + a_1 \times k_1 + \dots + a_s \times k_s$$

- C_{Tj} — transaction price for whole j-estate,
- S_i — the area of every i-element (parcel, parcel parts having defined soil classes, flat or building usable areas or whole building),
- c_i — i-element unit price,
- a_1, a_2, \dots, a_s — attributes accepted for urban estates estimation values,
- k_1, k_2, \dots, k_s — the weight coefficients of attributes accepted for urban estates valuation

VALUATION MODEL PARAMETERS' ESTIMATION

$$\begin{bmatrix} C_{T1} \\ C_{T2} \\ \dots \\ C_{Tj} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & K & S_{1i} \\ S_{21} & S_{22} & K & S_{2i} \\ \dots & \dots & \dots & \dots \\ M & M & K & M \\ \dots & \dots & \dots & \dots \\ S_{j1} & S_{j2} & K & S_{ji} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1s} \\ a_{21} & a_{22} & \dots & a_{2s} \\ \dots & \dots & \dots & \dots \\ a_{j1} & a_{j2} & \dots & a_{js} \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_i \\ \dots \\ k_1 \\ k_2 \\ \dots \\ k_s \end{bmatrix} + \begin{bmatrix} \delta_{T1} \\ \delta_{T2} \\ \dots \\ \delta_{Tj} \end{bmatrix}$$

VALUATION MODEL PARAMETERS' ESTIMATION

$$\{C_T\} = \{[S] \ [a]\} \cdot \begin{Bmatrix} [c] \\ [k] \end{Bmatrix} + \{\delta_T\}$$

- $\{C_T\}$ — transaction prices vector, for whole estates,
- $[S]$ — rectangular vertical matrix, containing areas of elements for elaborated estates
- $[a]$ — rectangular vertical matrix, containing attributes of elaborated urban estates
- $[c]$ — unit prices coefficients vector for estates' elements,
- $[k]$ — weight coefficients vector for elaborated attributes,
- $\{\delta_T\}$ — vector of random remainder (differences between transaction prices and model values).

VALUATION MODEL PARAMETERS' ESTIMATION

- The estimators of vector $[c]$ and vector $[k]$ may be written as following block matrices:

$$\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \begin{bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{bmatrix}^{-1} \times \begin{bmatrix} [S]^T \\ [a]^T \end{bmatrix} \times \{C_T\}$$

VALUATION MODEL PARAMETERS' ESTIMATION

$$\{\hat{\delta}_T\} = \{C_T\} - \{[S] \ [a]\} \cdot \begin{Bmatrix} \hat{c} \\ \hat{k} \end{Bmatrix}$$

$$\hat{\sigma}_0^2 = \frac{\{\hat{\delta}\}^T \{\hat{\delta}\}}{j - R\{[S] \ [a]\}}$$

VALUATION MODEL PARAMETERS' ESTIMATION

- The covariance matrix of estimated parameters may be written in formula

$$Cov \begin{Bmatrix} \hat{c} \\ \hat{k} \end{Bmatrix} = \hat{\sigma}_0^2 \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1}$$

VALUATION MODEL PARAMETERS' ESTIMATION

- After performing the analysis of variance for transaction prices vector for the whole estates, we may find the covariance matrix of $\{\hat{\delta}\}$ vector. It has the following form

$$Cov\{\hat{\delta}\} = \hat{\sigma}_0^2 \left\{ I - \{[S] \ [a]\} \times \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \{[S] \ [a]\}^T \right\}$$

THE VALUATION OF MARKET VALUE FOR URBAN ESTATES

- If we value the industrial estate, which include parcel and buildings of known areas, these quantities may be written as the one row parameters' matrix

$$[\bar{S}] = [\bar{S}_1 \ \bar{S}_2 \ \dots \ \bar{S}_i]$$

- The market attributes of valuated estate may be set together as the one row matrix, too

$$[\bar{a}] = [\bar{a}_1 \ \bar{a}_2 \ \dots \ \bar{a}_j]$$

THE VALUATION OF MARKET VALUE FOR URBAN ESTATES

- The estimated market value of elaborated urban estate can be expressed by the formula below

$$WR = \{[\bar{S}][\bar{a}]\} \times \begin{Bmatrix} \hat{c} \\ \hat{k} \end{Bmatrix}$$

- The variance of estimated market value for urbanized estate can be defined including covariance matrix as follows

$$\sigma^2(WR) = \{[\bar{S}][\bar{a}]\} \times Cov \begin{Bmatrix} \hat{c} \\ \hat{k} \end{Bmatrix} \times \begin{Bmatrix} [\bar{S}]^T \\ [\bar{a}]^T \end{Bmatrix} \times \{[\bar{S}][\bar{a}]\}^T$$

THE EXAMPLES OF URBAN ESTATES' VALUATION

The application of valuation parametric model will be presented on the example of urban estates. Information from seven authenticated deeds obtained from public notaries was used here.

The following attributes has been taken into account during calculations:

- part of the city - suburbs,
- destination in land use city plan – the land of low urbanization intensification,
- localization – very good (2), good (1),
- access to parcel (estate)– good,
- vicinity – very good,
- public utilities – water, electricity, gas, sewage, road,
- standard of components used for building and decorating (building standard) – very high (2), high (1),
- usable area of urbanized buildings from 200 m² to 340 m²,
- area of parcel from 810 m² to 1050 m².

THE EXAMPLES OF URBAN ESTATES' VALUATION

There are seven urban real estates that were chosen for consideration. They description and attributes are listed below.

1. Built up estate with commercial building. Building usable area is 260 m². Parcel area is 850 m². Attributes: localization – very good (2), building standard – high (1), price – 1 570 000 zł (402 000 EUR 1 EUR = 3.9 zł)
2. Built up estate with commercial building. Building usable area is 300 m². Parcel area is 970 m². Attributes: localization – good (1), building standard – high (1), price – 1 600 000 zł (410 000 EUR)
3. Built up estate with commercial building. Building usable area is 220 m². Parcel area is 760 m². Attributes: localization – very good (2), building standard – very high (2), price – 1 450 000 zł (372 000 EUR)
4. Built up estate with commercial building. Building usable area is 320 m². Parcel area is 910 m². Attributes: localization – very good (2), building standard – high (1), price – 1 800 000 zł (462 000 EUR)
5. Built up estate with commercial building. Building usable area is 200 m². Parcel area is 810 m². Attributes: localization – good (1), building standard – high (1), price – 1 200 000 zł (308 000 EUR)
6. Built up estate with commercial building. Building usable area is 340 m². Parcel area is 1050 m². Attributes: localization – very good (2), building standard – high (1), price – 1 900 000 zł (487 000 EUR)
7. Built up estate with commercial building. Building usable area is 290 m². Parcel area is 880 m². Attributes: localization – good (1), building standard – high (1), price – 1 570 000 zł (397 000 EUR).

The estimation of valuation model's parameters

- The information obtained from deeds is basis for 7 equations written according to the formulas presented before. Every equation has two price coefficients and two weight coefficients. The matrix [S] of parcel areas and building usable areas and the matrix [a] of market attributes and the matrix {C_r} of transaction prices have the following form:

$$[S] = \begin{bmatrix} 850 & 260 \\ 970 & 300 \\ 760 & 220 \\ 910 & 320 \\ 810 & 200 \\ 1050 & 340 \\ 880 & 290 \end{bmatrix} \quad [a] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 1 \\ 1 & 1 \\ 2 \\ 1 & 1 \end{bmatrix} \quad \{C_r\} = \begin{bmatrix} 1570000 \\ 1600000 \\ 1450000 \\ 1800000 \\ 1200000 \\ 1900000 \\ 1550000 \end{bmatrix}$$

The estimation of valuation model's parameters

From the estimation of this model's parameters according to formulas presenting unit prices coefficients and random remainders calculated for every transaction, were obtained. The unit parcel and urban buildings' prices coefficients are as follows:

$$\hat{c}_G = 349,43 \text{ zł} / \text{m}^2$$

$$\hat{c}_B = 3547,27 \text{ zł} / \text{m}^2$$

The estimation of valuation model's parameters

- The covariance matrix obtained for all estimated parameters consists of the following values

$$\text{Cov} \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = 1657,983 \begin{bmatrix} 31.02 & -92.34 & 1561.85 & -3856.94 \\ /- / & 295.00 & -7277.59 & 10584.97 \\ /- / & /- / & 898922.81 & -695651.17 \\ /- / & /- / & /- / & 1424580.04 \end{bmatrix}$$

The valuation of similar estates

- As a subject of valuation we choose estate similar to estates elaborated in market analysis presented above. This is the commercial estate, that has usable area of 260 m². This parcel has an area of 980 m². The localization is good (1) and building standard high (1). Applying formula we obtained estates market value stands as follows:

$$WR = \begin{bmatrix} [S] \\ [a] \end{bmatrix} \times \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \begin{bmatrix} 980 & 260 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 349,43 \\ 3547,27 \\ 137789 \\ 65959 \end{bmatrix} = 1\,468\,480 \text{ zł}$$

The valuation of similar estates

- After performing operation on the proper matrices, the standard deviation of estimated market value was calculated. It is presented below:

$$\sigma^2(WR) = 1657,983 \times \begin{bmatrix} 980 & 260 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 31.02 & -92.34 & 1561.85 & -3856.94 \\ /- / & 295.00 & -7277.59 & 10584.97 \\ /- / & /- / & 898922.81 & -695651.17 \\ /- / & /- / & /- / & 1424580.04 \end{bmatrix} \times \begin{bmatrix} 980 \\ 260 \\ 1 \\ 1 \end{bmatrix}$$

$$\sigma^2(WR) = 1377460000 \Leftrightarrow \sigma(WR) = 37\,114 \text{ zł}$$

The valuation of similar estates

- After applying Student distribution quantile, the symmetric confidence intervals for estimated estate's market value equal

$$\hat{WR} = 1\,468\,480 \pm 118\,765 z_t$$

Thank you