Attitude Determination by Means of Dual Frequency GPS Receivers

Vadim Rokhlin and Gilad Even-Tzur
Department of Mapping and Geo-Information Engineering
Faculty of Civil and Environmental Engineering
Technion - Israel Institute of Technology

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Introduction

- Attitude determination in air, sea or land with high accuracy and high reliability has always been a technological and engineering challenge.
- Attitude determination can contribute to improving civil and military systems.
- Using GPS technology is not requiring pre-calibration as needed when using IMU.

Introduction (cont.)

- The basic idea behind the attitude determination based on a GPS multi-antenna system is to calculate first the baselines between the antennas using differential positioning and then to derive the attitude parameters.
- For high-accuracy applications, carrier phase data should be employed after the integer cycle ambiguities are correctly resolved.
- Processing only the pseudorange measurements also yields the attitude parameters, but with lower accuracy.

Purpose

The purpose of the current research is to examine speed of solution from dual frequency GPS measurements against one frequency receiver and examine L1 and L2 combination on the solution of the attitude parameters.

Coordinate System

Two coordinate Systems need to be distinguished:

- The local level coordinate frame - LLF.
- The antenna body frame - ABF.

Attitude Determination

• Attitude of a rigid body platform is determined by the orientation of the specified body frame coordinate system with respect to the reference coordinate system.

• By using a GPS multi-antenna system, the attitude of the GPS antenna body frame with respect to the local level frame can be precisely computed at each observation epoch.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{ABF}^{b} = R_2(r)R_1(p)R_3(h)\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{LLF}^{l}$$

Attitude Determination (cont.)

There are two methods for estimation of heading, pitch and roll :

- Direct computation method which does not require knowledge of the antenna's body frame coordinates and only uses the local level coordinates of the three antennas that define the platform.
- Least squares estimation procedure that make full use of all the position information of multiple antennas in the local level as well as in body frame.

Direct Attitude Determination Approach

- Two antennas determine the Azimuth (h) and pitch (p) of the platform.

$$h = -\tan^{-1}(X_{2,l} / Y_{2,l})$$

$$p = \tan^{-1}(Z_{2,l} / \sqrt{X_{2,l}^2 + Y_{2,l}^2})$$

- Roll (r) parameter is obtained after rotation coordinates of third antenna in LLF system by heading and pitch angles and transforming them to ABF system.

$$r = -\tan^{-1}(Z''_{3,l} / X''_{3,l})$$

Least Squares Estimation

- When we have more than three antennas LSE gives the best estimates based on all the position information contained in a multiple GPS antenna array.

- The rotation matrix R is solely defined by the three elements, h, p and r.

- For each antenna the following relation is obtained:

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}^b_{ABF} = R_2(r)R_1(p)R_3(h)\begin{pmatrix} X_{i,l} \\ Y_{i,l} \\ Z_{i,l} \end{pmatrix}_{LLF} \quad , when \quad i = 2,3 \ldots n$$

Double Difference Method of GPS Carrier Phase

Double Difference equation:

$$\Phi^{jk}_{AB,L_1/L_2}(t) = \rho^{jk}_{AB}(t) + \lambda N^{jk}_{AB,L_1/L_2} - \ldots$$

$$I^{jk}_{AB,L_1/L_2} + T^{jk}_{AB} + noise$$

Double Difference Method of GPS Carrier Phase with Dual Frequency Receiver

Four satellites j, k, l, m and two epochs t_1, t_2 and the system follows as:

$$\ell_{L1} = \begin{bmatrix} \ell^{jk}_{AB,L1}(t_1) \\ \ell^{jl}_{AB,L1}(t_1) \\ \ell^{jm}_{AB,L1}(t_1) \\ \ell^{jk}_{AB,L1}(t_2) \\ \ell^{jl}_{AB,L1}(t_2) \\ \ell^{jm}_{AB,L1}(t_2) \end{bmatrix}; \ell_{L2} = \begin{bmatrix} \ell^{jk}_{AB,L2}(t_1) \\ \ell^{jl}_{AB,L2}(t_1) \\ \ell^{jm}_{AB,L2}(t_1) \\ \ell^{jk}_{AB,L2}(t_2) \\ \ell^{jl}_{AB,L2}(t_2) \\ \ell^{jm}_{AB,L2}(t_2) \end{bmatrix}; x = \begin{bmatrix} dX_B \\ dY_B \\ dZ_B \\ N^{jk}_{AB,L1} \\ N^{jl}_{AB,L1} \\ N^{jm}_{AB,L1} \\ N^{jk}_{AB,L2} \\ N^{jl}_{AB,L2} \\ N^{jm}_{AB,L2} \end{bmatrix}$$

$$\ell = \begin{bmatrix} \ell_{L1} \\ \ell_{L2} \end{bmatrix} = Ax;\; x = (A^T P A)^{-1} A^T P \ell$$

Double Difference Method of GPS Carrier Phase with Dual Frequency Receiver (cont.)

$$\ell_{L1,L2} = Ax = [A_a \quad A_\lambda]x = A_a \begin{bmatrix} dX_B \\ dY_B \\ dZ_B \end{bmatrix} + A_\lambda \begin{bmatrix} N^{jk}_{AB,L1} \\ N^{jl}_{AB,L1} \\ N^{jm}_{AB,L1} \\ N^{jk}_{AB,L2} \\ N^{jl}_{AB,L2} \\ N^{jm}_{AB,L2} \end{bmatrix}$$

$$A_\lambda = [A_{\lambda 1} \quad A_{\lambda 2}]$$

$$A_a = \begin{bmatrix} A_{a1} \\ A_{a2} \end{bmatrix}$$

Least-square AMBiguity Decorrelation Adjustment (LAMBDA) method

- The parameter estimation is carried out in three steps:
  1. float solution.
  2. integer ambiguity estimation.
  3. fix solution .

- Least-squares principle used to compute the integer difference ambiguities and the baseline coordinates:

$$\min_{a,b} \left\| y - A_\lambda a - A_\alpha b \right\|^2_{Q_y} \quad , a \in Z^n , b \in R^p$$

RECURSIVE LEAST SQUARE ADJUSTMENT

- New measurements are added after every new epoch.
- The measurements are included into the least square along with the previous solution, while taking into account the cofactor matrix of the previous solution.
- Every new epoch is contributing for more accurate value of the variables.

Linear Combination Of The Frequencies

- The most commonly used Linear Combinations when working with the original L_1 and L_2 frequencies:

Symbol	a1	a2	$\lambda_{a1,a2}$	Remark
L1	1	0	19.0	Original L_1 signal
L2	0	1	24.4	Original L_2 signal
Lc	1	f_{L2} / f_{L1}	48.4	Ionosphere - free, floating ambiguities
Lw	1	-1	86.2	Wide lane
Ln	1	1	10.7	Narrow lane

$$\lambda_{a1,a2} = \frac{c}{a_1 \cdot f_{L1} + a_2 \cdot f_{L2}}; \quad f_{a1,a2} = \frac{c}{\lambda_{a1,a2}} = a_1 \cdot f_{L1} + a_2 \cdot f_{L2}$$

Linear Combination (cont.)

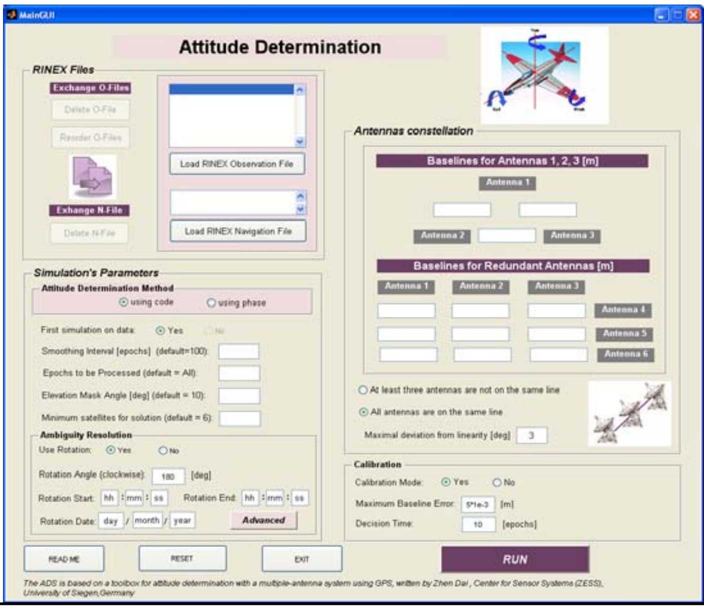
- Combined double difference phase measurement equation :

$$\Phi^{ij}_{AB[a1,a2]} = a_1 \cdot \Phi^{ij}_{AB[L1]} + a_2 \cdot \Phi^{ij}_{AB[L2]}$$

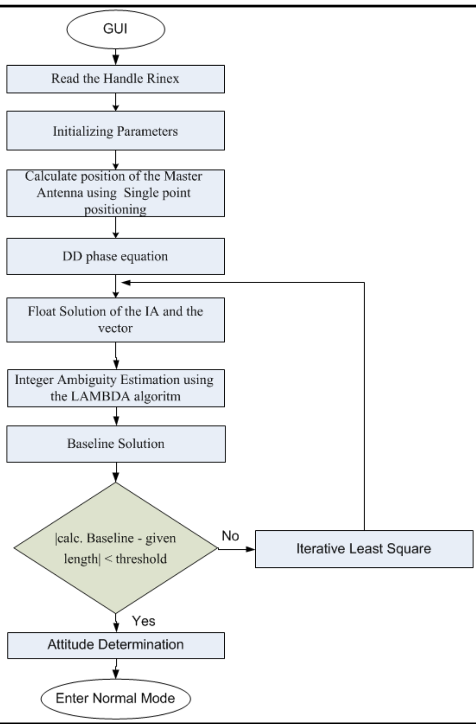
- Ambiguity associated with the linearly combined phase measurement:

$$N^{ij}_{AB[a1,a2]} = a_1 \cdot N^{ij}_{AB[L1]} + a_2 \cdot N^{ij}_{AB[L2]}$$

GUI



Block Diagram

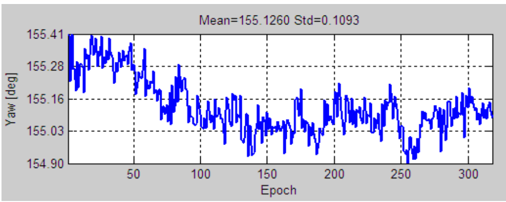


Test with two Dual Frequency Antennas

Two dual frequency Ashtech Z-Surveyor GPS receivers were used to collect 80 minutes of data at 1 second interval simultaneously in semi-static mode. 8 satellites were used for the solution of the observation equation, and the Pdop values ranged between 1.8 and 2.1.

Test Results – Static Mode

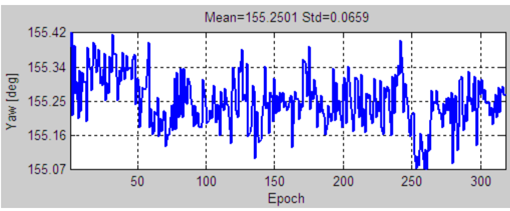
L1:



Number of epochs = 200

Azimuth mean = 155.12 °
Std = 0.11 °

L1L2:

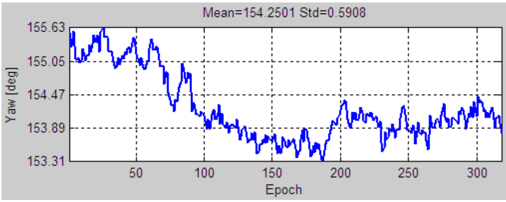


Number of epochs = 40

Azimuth mean = 155. 25 °
Std = 0.07 °

Test Results – Static Mode

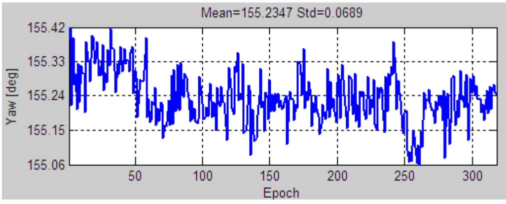
Lw:



Number of epochs = 100

Azimuth mean = 154.25 °
Std = 0.6 °

Ln:

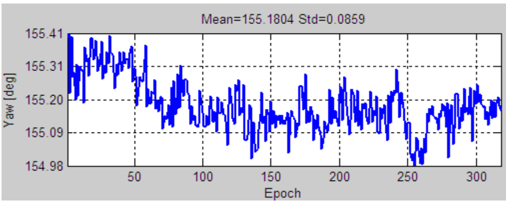


Number of epochs =225

Azimuth mean = 155.23 °
Std = 0.07 °

Test Results – Static Mode

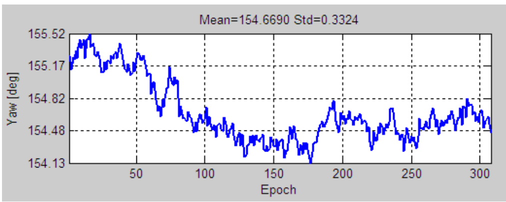
L1Ln:



Number of epochs = 80

Azimuth mean = 155.18 °
Std = 0.09 °

L1Lw:

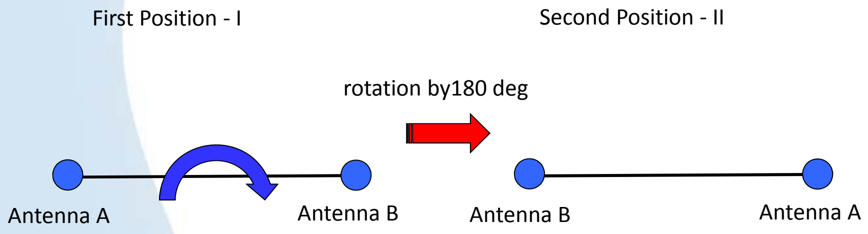


Number of epochs = 20

Azimuth mean = 154.67 °
Std = 0.33 °

Baseline Rotation Method

- Antenna swap method:



The vector is:

$$(dX, dY, dZ)_I$$

The vector is:

$$(dX, dY, dZ)_{II} = (-dX, -dY, -dZ)_I$$

Baseline Rotation Method (cont.)

First position- I:

$$\ell_{I_{L1,L2}} = A_{\alpha I}\begin{bmatrix} dX_B \\ dY_B \\ dZ_B \end{bmatrix} + A_\lambda \begin{bmatrix} N^{jk}_{AB,L1} \\ N^{jl}_{AB,L1} \\ N^{jm}_{AB,L1} \\ N^{jk}_{AB,L2} \\ N^{jl}_{AB,L2} \\ N^{jm}_{AB,L2} \end{bmatrix}$$

Second position- II:

$$\ell_{II_{L1,L2}} = A_{\alpha II}\begin{bmatrix} -dX_B \\ -dY_B \\ -dZ_B \end{bmatrix} + A_\lambda \begin{bmatrix} N^{jk}_{AB,L1} \\ N^{jl}_{AB,L1} \\ N^{jm}_{AB,L1} \\ N^{jk}_{AB,L2} \\ N^{jl}_{AB,L2} \\ N^{jm}_{AB,L2} \end{bmatrix}$$

Baseline Rotation Method (cont.)

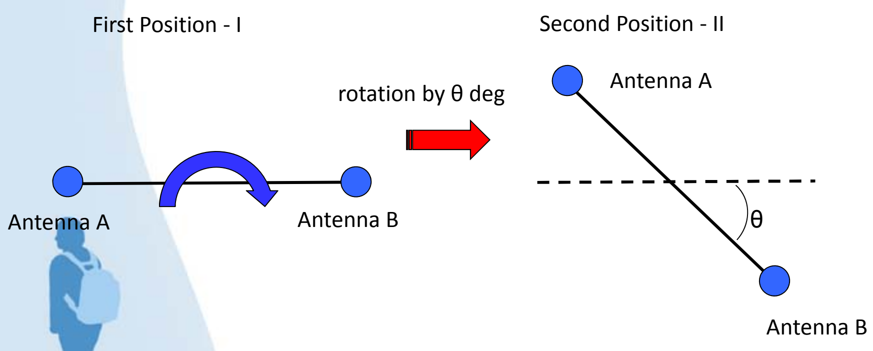
Difference between equations:

$$\ell_{II} - \ell_{I} = -(A_{aII} + A_{aI}) \begin{bmatrix} dX_B \\ dY_B \\ dZ_B \end{bmatrix}$$

As a result, the true vector can be obtained without solving for the IA variables.

Baseline Rotation Method (cont.)

Antenna swap by θ deg :



In this case:

$$(dX, dY, dZ)_I \neq (dX, dY, dZ)_{II}$$

Baseline Rotation Method (cont.)

Antenna swap by θ deg:

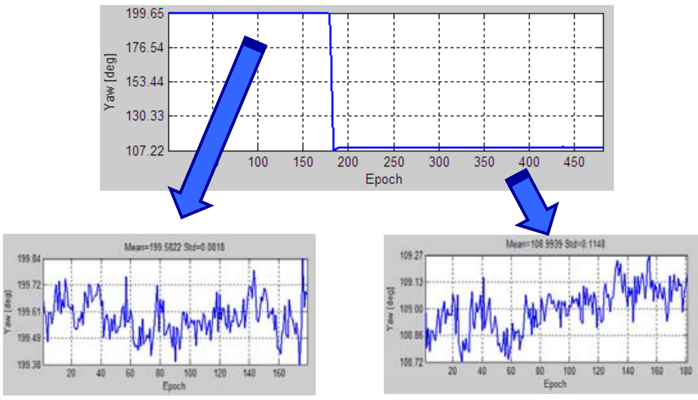
$$\ell_{II} - \ell_{I} = (A_{aII} - A_{aI} P_I^{-1} R_3(\theta)^T P_{II}) \begin{bmatrix} dX_B \\ dY_B \\ dZ_B \end{bmatrix}_{II}$$

$R_3(\theta)$ – rotation matrix about z-axis.

P - transformation matrix between Cartesian and LLF coordinate systems.

Test Results with Antenna Swap

L1:



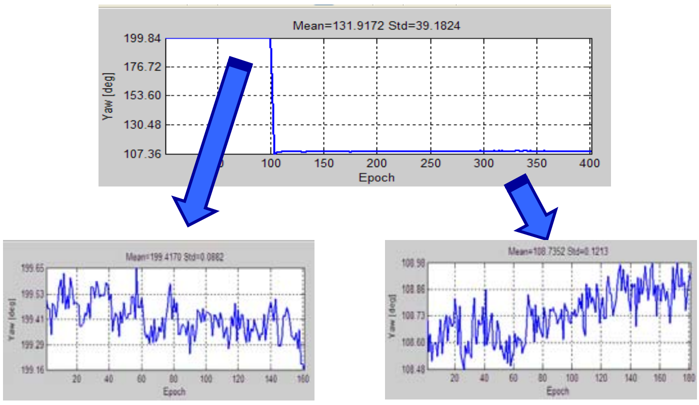
Azimuth mean = 199.58 º
Std = 0.08 º

Azimuth mean = 108.99 º
Std = 0.11 º

Number of epochs = 90+50+90 = 230 sec

Test Results with Antenna Swap

L1 L2:



Azimuth mean = 199.42 º
Std = 0.09 º

Azimuth mean = 108.74 º
Std = 0.12 º

Number of epochs =20+30+20 = 70 sec

Conclusions

- Combining different frequencies significantly shortens the solution's time of the integer number of the wavelengths.
- In less than 30 seconds a solution can be obtained once the receiver starts picking up the satellites signals.
- Number of frequencies have no dramatic influence on the solution's accuracy .

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Thank you !

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